

Statistical Models for Jupiter's Troposphere and the $\text{NH}_3\text{-H}_2\text{O}$ Cloud Systems

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A knowledge of planetary atmospheric uncertainties is required for the design of entry probes. Examination of the atmospheric properties of Jupiter suggests an adiabatic lapse rate. Monte Carlo calculations are used to determine probability distributions in the temperature-pressure-density profiles. Once a temperature-pressure model is constructed the ammonia concentration in the atmosphere is estimated from observed brightness temperature data. The atmospheric water concentration is estimated from corrected solar abundance ratios. The $\text{H}_2\text{O-NH}_3$ cloud structures are then determined. Knowledge of these cloud structures is necessary for analyzing science mission objectives and is important for sizing the spacecraft communications system.

Nomenclature

C	= polytropic parameter
C_p, C_v	= mean molar specific heats at constant pressure and volume, respectively
C_H, C_{He}	= molar specific heats at constant pressure for hydrogen and helium, respectively
D	= cloud density
g	= gravitational acceleration at altitude z
$\bar{\mu}$	= mean atmospheric molecular weight
μ_H, μ_{He}	= molecular weights of hydrogen and helium, respectively
P	= total atmospheric pressure
R	= universal gas constant
ρ	= atmospheric density
T	= atmospheric temperature
X_a, X_w	= mass fractions of ammonia and water in the atmosphere, respectively
Z	= altitude—zero altitude is defined to be at 125°K

Introduction

ATMOSPHERIC models that accurately describe our present knowledge of planetary uncertainties are required for entry probe design. Properties of Jupiter's atmosphere are presented in the "Handbook of the Physical Properties of the Planet Jupiter"¹ and in the NASA SP-8069 monograph.² It is the author's belief that the limiting atmospheric models ("cool" and "warm" models) presented in the NASA SP-8069 monograph are unrealistic. The purpose of this paper is to present a method for modeling these limiting atmospheres.

In developing atmospheric models, one of the greatest problems is determining the temperature lapse rate at different depths in the atmosphere. Stone³ has published an article which indicates that the temperature lapse rate of Jupiter's troposphere lies well within the uncertainties of the adiabatic lapse rate. This paper presents a method for determining the uncertainties in the parameters that define a hydrostatic-adiabatic atmosphere.

Atmospheric radio attenuation is sensitive to the atmospheric ammonia concentration and cloud structures. Richardson⁴ has developed a method for determining the atmospheric ammonia concentration for a particular temperature-pressure profile from the available brightness temperature data in the 5 cm–30 cm wavelength region. Once the temperature-pressure profile and ammonia concentrations are known, ammonia-water cloud models are determined by using Lewis's⁵ method.

Temperature Lapse Rate

A planet's temperature lapse rate may be either adiabatic (neutral stability), subadiabatic (stable against convection), or superadiabatic (convectively unstable). Atmospheric parcels may lose energy primarily by radiation, convection, or a combination of both. If Jupiter is assumed to be in radiative equilibrium, then the amount of energy an atmospheric parcel receives is equal to the amount of energy that it radiates. Trafton⁶ has performed a detailed analysis of this problem and concludes that the radiative equilibrium temperature lapse rate is -5°K/km . The adiabatic temperature lapse rate is

$$(\partial T / \partial Z) = -\bar{\mu}g/C_p = -1.9 \pm 0.3 (^\circ\text{K/km}) \quad (1)$$

Since $|\partial T / \partial Z|_e > |\partial T / \partial Z|_a$ where e denotes radiative equilibrium and a denotes adiabatic equilibrium, the atmosphere will be convective. (The Schwarzschild inequality is satisfied.)

Can the atmosphere actually become subadiabatic? The answer is yes. In order for the atmosphere to become subadiabatic, a driving mechanism is required to overcome the stratification forces. One such driving mechanism is in the form of horizontal temperature gradients. (The poles are colder than the equator.) The resulting baroclinic motions may be Hadley cells, cyclones, or anticyclones.

These baroclinic phenomena have been observed in the form of dark and light spots on Jupiter, cyclones and anticyclones, respectively.¹ Westphal⁷ has observed in the 5μ region that the dark bands on Jupiter appear warmer than the light colored zones and has concluded that the dark bands are the absence of ammonia clouds and the light zones are ammonia clouds. This may be interpreted as downward motion in the bands and upward motion in the zone (Hadley cells). Ingersoll and Cuzzi⁸ have used this Hadley model along with the assumption of geostrophic equilibrium (horizontal pressure gradients are balanced by Coriolis forces) to account for the differential rotational periods between adjacent zones.

Thus, there is strong evidence to support the theory that baroclinic motion is important. Stone³ has carefully studied the effects of baroclinic circulation on Jupiter and has concluded that the actual time averaged temperature lapse rate is subadiabatic and deviates from the adiabatic lapse rate value by about $10^{-3} ^\circ\text{K/km}$. Since Jupiter's atmosphere deviates only slightly from an adiabatic atmosphere, it is valid to model Jupiter's atmosphere as though it is adiabatic.

Tropospheric Models

Engineering atmospheric models should provide an adequate representation of the state of knowledge of the atmosphere. Since Stone³ has appreciably reduced the uncertainty of Jupiter's

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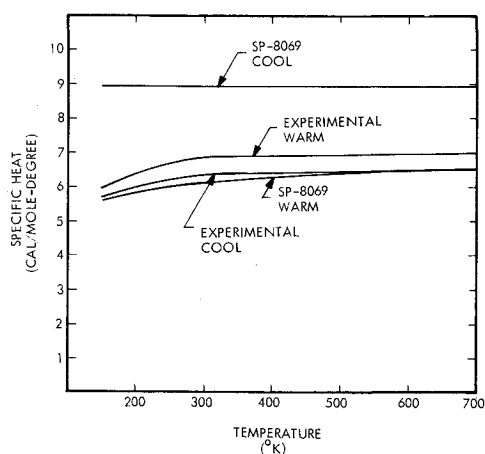


Fig. 1 Comparison of the specific heat values used in the NASA SP-8069 document with experimental specific heat values.

atmosphere, this paper presents a method for updating the present NASA SP-8069 atmospheric models for spacecraft design.

The atmospheric models presented in the NASA document were identified as "nominal," "cool," and "warm." The cool atmosphere is characterized by low temperature, high pressure, and high density; the warm atmosphere is characterized by high temperature, low pressure, and low density. The same nomenclature is used in this paper.

The relationship of temperature, pressure and density is given by

$$\rho = (\bar{\mu}P/RT) \quad (2)$$

In order to maximize the density one would normally choose $\bar{\mu}$ and P to be a maximum and T minimum as was done in the NASA document.² The adiabatic temperature lapse rate is given by

$$(\partial T/\partial Z) = -(\bar{\mu}g/C_p) \quad (3)$$

Since the reference temperature is located near the top of the troposphere, Eq. (3) is used to find the temperature for depths deeper in the atmosphere. Therefore, a maximum value for $\bar{\mu}$ corresponds with a maximum value for the temperature. This situation is in conflict with predictions of maximum and minimum values for the density. The procedure used in the NASA document to avoid this conflict was to assign specific heat values independently of composition. Figure 1 compares the specific heat values used in the NASA document with experimental specific heat values. The curves in Fig. 1 marked

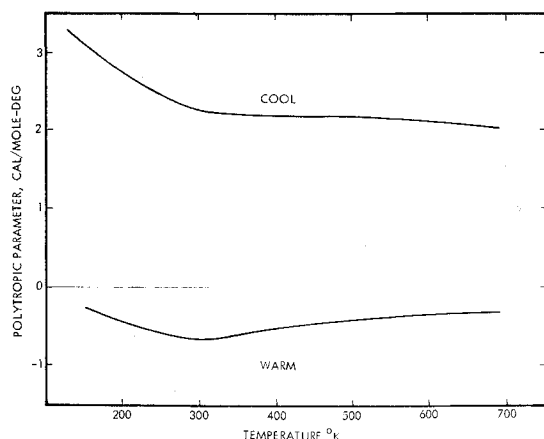


Fig. 2 Polytropic parameter vs temperature. The polytropic parameters were determined from the atmospheric models presented in the NASA SP-8069 document.

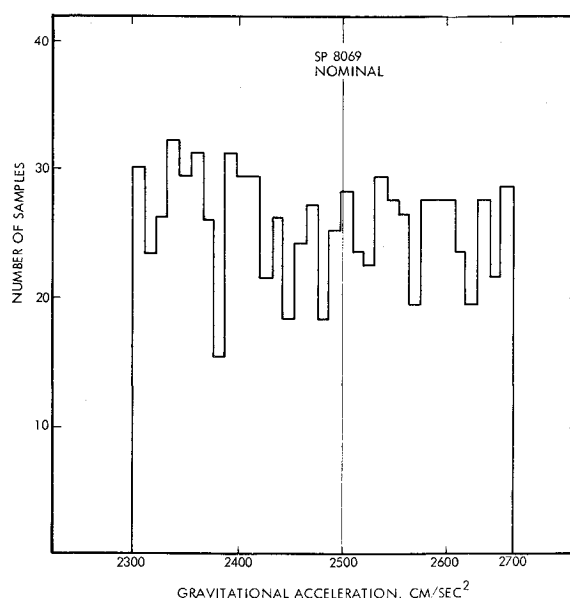


Fig. 3 The random distribution of gravitational acceleration values used for determining the temperature-pressure-density profiles.

"warm" and "cool" are the experimental specific heat values for the respective "warm" and "cool" atmospheric compositions as defined in the NASA SP-8069 document.

The author⁹ of the NASA SP-8069 document did not intend to treat Jupiter's atmosphere nonadiabatically; but, assigning specific heat values independently of composition is equivalent to treating the atmosphere nonadiabatically. For a nonadiabatic atmosphere

$$dQ = C_p dT + PdV \quad (4)$$

where dQ is the amount of energy absorbed or radiated by a parcel and can be approximated by $dQ = -CdT$. An atmosphere that behaves in this manner is referred to as a polytropic atmosphere. With the ideal gas and hydrostatic assumptions, the lapse rate is given by

$$(\partial T/\partial Z) = -(\bar{\mu}g/C_p + C) \quad (5)$$

where $C = 0$ for an adiabatic atmosphere, C is positive for a subadiabatic atmosphere, and C is negative for a superadiabatic atmosphere. The values for C vs temperature used in the NASA models are plotted in Fig. 2.

Since the analytic method for determining the temperature, pressure, and density probability distributions within the adiabatic constraints is quite difficult, a Monte Carlo method is suggested. The temperature-pressure profiles are calculated using Eq. (5) and the hydrostatic equation for an ideal gas

$$(\partial P/\partial Z) = -(\bar{\mu}gP/RT) \quad (6)$$

The uncertainty limits for the parameters are listed in Table 1. The values for the mean molecular weight and the reference pressure are the same as those found in the NASA document.² The maximum value for gravitational acceleration is at the poles and the minimum value is the lowest expected value at the equator. The uncertainties for all of these parameters are nongaussian since they have definite bounds (the mean molecular

Table 1 Uncertainty limits for the Jovian atmosphere parameters

Parameters	Max	Min
Mean molecular weight (gm/mole)	2.7	2.14
Gravitational acceleration (cm/sec ²)	2700	2300
Pressure at the reference temperature of 125°K (atm)	0.5	0.2

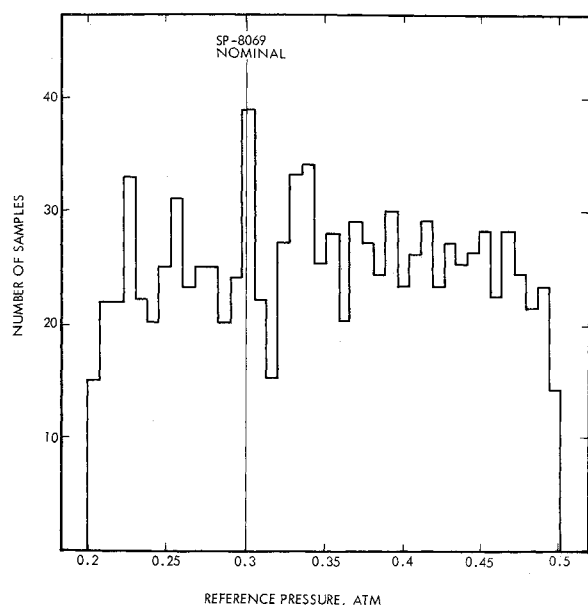


Fig. 4 The random distribution of reference pressure values used for determining the temperature-pressure-density profiles.

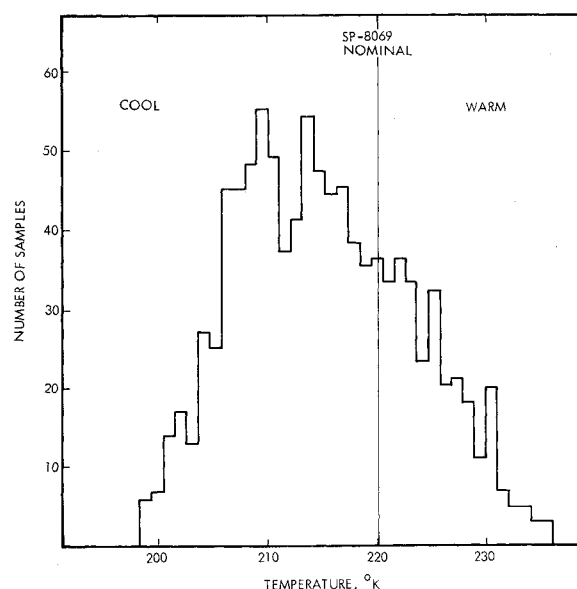


Fig. 6 The temperature probability distribution at 40 km below the 125°K reference point.

weight cannot be less than one; the pressure cannot be negative, etc.). For this paper the atmospheric parameters are assumed to be constrained within the bounds described in Table 1.

Histograms of the atmospheric parameters resulting from 999 samples are presented in Figs. 3-5. Equal probability was assigned to all values of the reference pressure because of our lack of confidence in determining a reference pressure. (Calculations for determining the reference pressure are dependent upon the atmospheric radiative transfer properties which are quite uncertain.) The distribution for the mean molecular weight was weighted in favor of a hydrogen-rich atmosphere because most theorists agree that Jupiter's elemental relative abundances should approximate that of the sun.

Figures 6-11 are histograms which represent the temperature, pressure, and density distributions at atmospheric depths of 40 and 200 km. The reference altitude (zero altitude) is arbitrarily

defined to be that altitude with a temperature of 125°K. These distributions were calculated from Eqs. (2, 3, and 5) using the input distributions illustrated in Figs. 3-5. For each value of the mean molecular weight chosen in the Monte Carlo process, the specific heat was calculated from the following expression:

$$C_p = \frac{\mu_{\text{He}} - \bar{\mu}}{\mu_{\text{He}} - \mu_{\text{H}}} C_{\text{H}} + \frac{\bar{\mu} - \mu_{\text{H}}}{\mu_{\text{He}} - \mu_{\text{H}}} C_{\text{He}} \quad (7)$$

In Figs. 12-16 the temperature-pressure-density profiles resulting from the Monte Carlo analysis are compared with data presented in the NASA SP-8069 monograph. The "max warm" and "max cool" curves correspond to the limiting cases predicted by the Monte Carlo analysis.

A comparison of the $\text{H}_2\text{O}-\text{NH}_3$ cloud structures resulting from the following analysis with the cloud structures presented in the NASA SP-8069 monograph is presented in Fig. 17. In

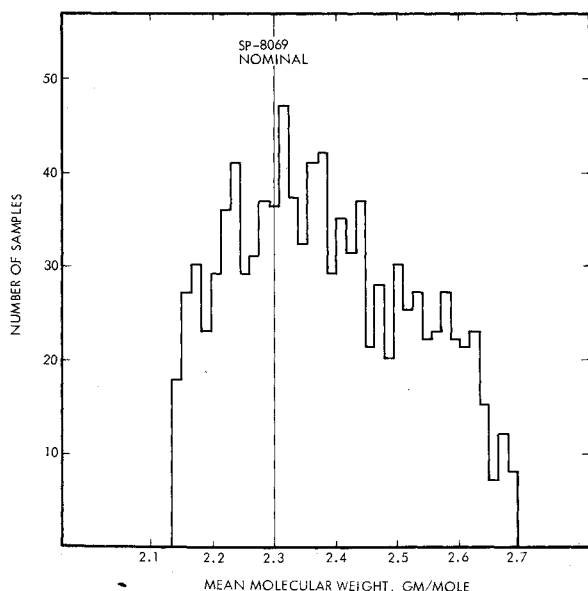


Fig. 5 A random truncated gaussian distribution of mean molecular weight values about the nominal value used for determining the temperature-pressure-density profiles.

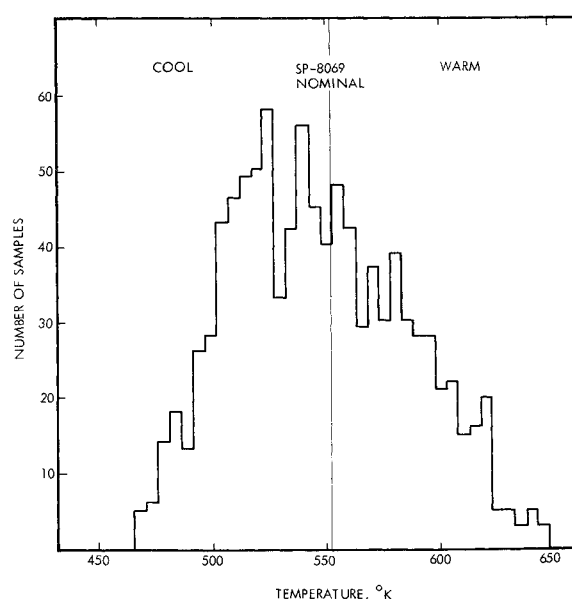


Fig. 7 The temperature probability distribution at 200 km below the 125°K reference point.

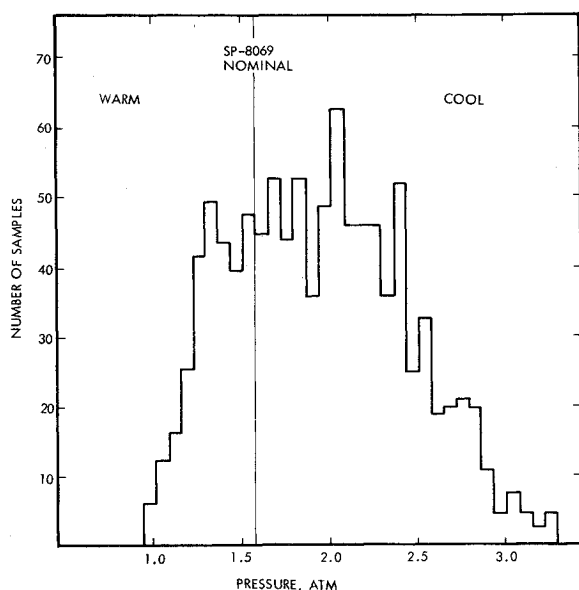


Fig. 8 The pressure probability distribution at 40 km below the 125°K reference point.

both cases the theory used for predicting the cloud structures is that of John S. Lewis.⁵ The expression for the cloud density used for this analysis is

$$D = -\frac{P}{g} \left(\frac{\partial X_a}{\partial Z} + \frac{\partial X_w}{\partial Z} \right) \quad (8)$$

The amount of atmospheric radio attenuation is a function primarily of the atmospheric ammonia concentration and the temperature-pressure profile. The ammonia abundance in the lower atmosphere is determined from brightness temperature measurements using Richardson's⁴ method with the results presented in Figs. 18 and 19. The atmospheric water concentration is estimated from the corrected solar abundance ratios.⁵ The condensation rate $[\partial(X_a + X_w)/\partial Z]$ is determined from Haundenschild's¹⁰ semiempirical equations. The resulting atmospheric radio attenuation for a 16 GHz signal is plotted in Fig. 20.

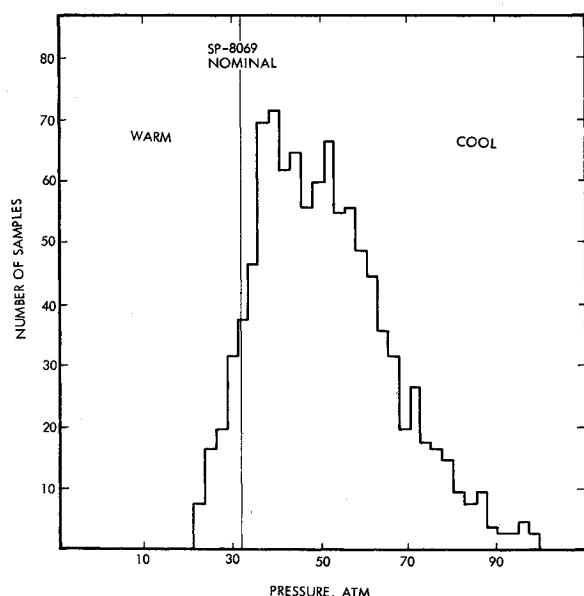


Fig. 9 The pressure probability distribution at 200 km below the 125°K reference point.

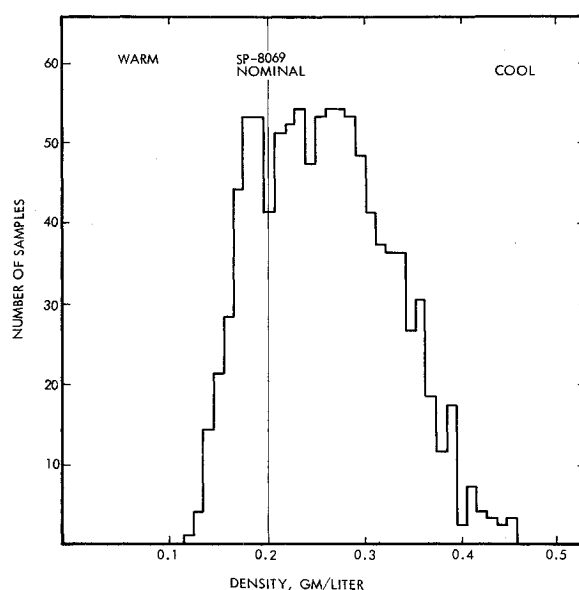


Fig. 10 The density probability distribution at 40 km below the 125°K reference point.

The uncertainty associated with modeling the Jovian cloud distribution can be analyzed in two parts: that associated with the Jovian atmospheric models (Fig. 17) and that associated with vertical winds that can disperse the clouds over large altitude regions. The latter condition may be analyzed by comparing Earth with Jupiter. On Earth we observe the atmosphere to be highly convective; thus the vertical wind velocities distribute the clouds over a large altitude region. On Jupiter this should not occur. By comparing Earth with Jupiter we see that the Sun deposits most of its energy on the ground which radiates in the infrared thus heating the lower atmosphere causing large vertical wind velocities. Since on Jupiter the solar energy is deposited over a large region in the Jovian atmosphere, the resulting temperature gradients are closer to adiabatic resulting in smaller vertical wind velocities. The specific heat (erg/gm-deg) of Jupiter's atmosphere is about an order of magnitude larger than Earth's; therefore, the same energy is

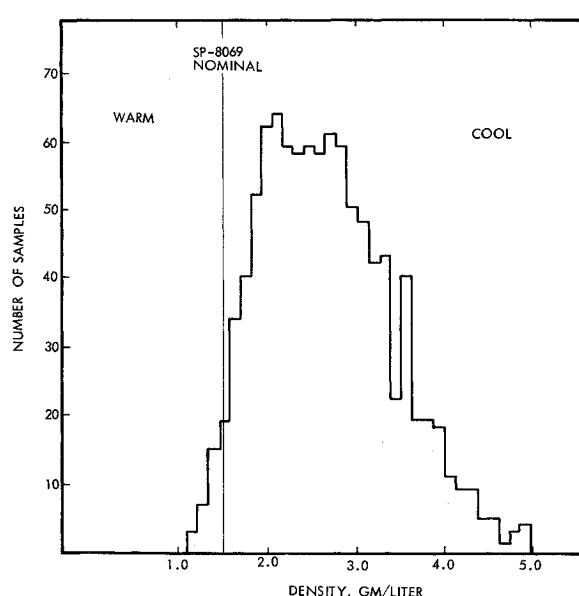


Fig. 11 The density probability distribution at 200 km below the 125°K reference point.

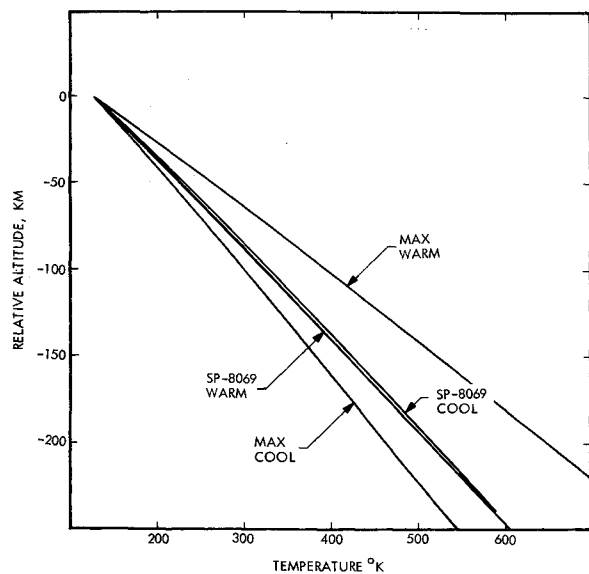


Fig. 12 Relative altitude vs temperature.

convected with smaller wind velocities, and Jupiter is further away from the sun than Earth so less energy is available for heat convection. We can, therefore, conclude that uncertainties arising from the vertical wind velocities in Jupiter's atmosphere

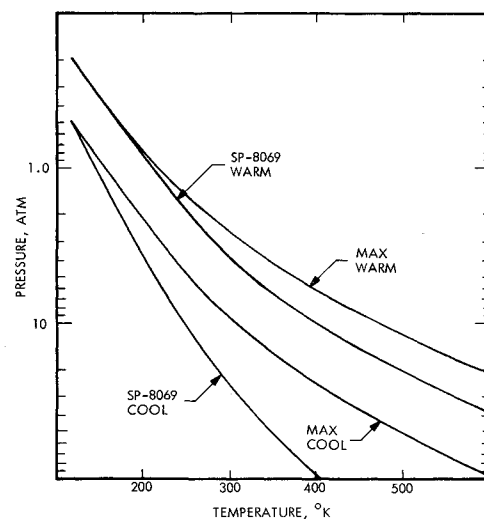


Fig. 15 Temperature-pressure profiles.

can be neglected when compared with uncertainties arising from the atmospheric models.

Conclusions

Results of this study suggest that in developing engineering models for Jupiter's troposphere consideration be given to the following conclusions.

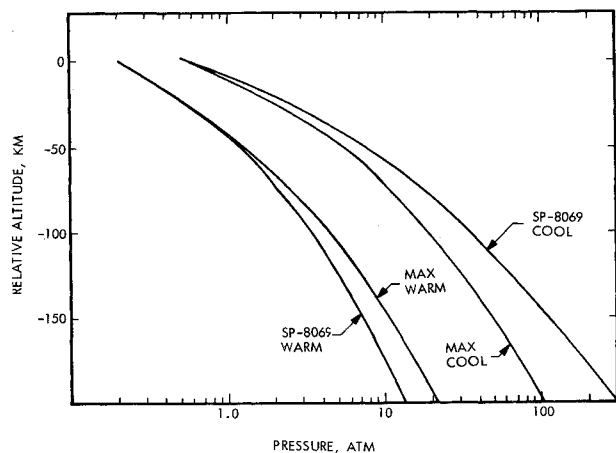


Fig. 13 Relative altitude vs pressure.

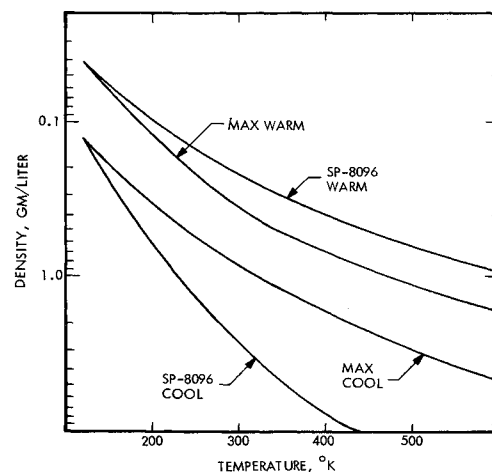


Fig. 16 Temperature-density profiles.

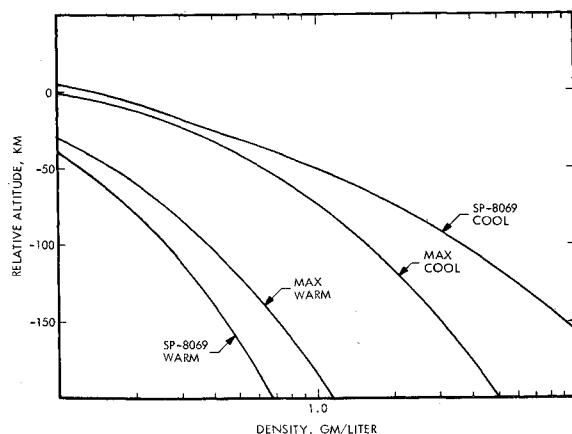


Fig. 14 Relative altitude vs density.

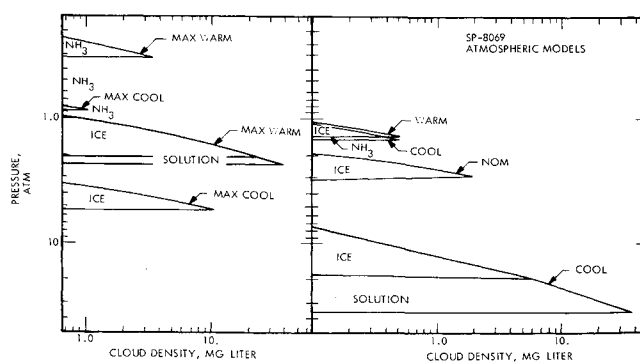


Fig. 17 Atmospheric pressure vs cloud density.

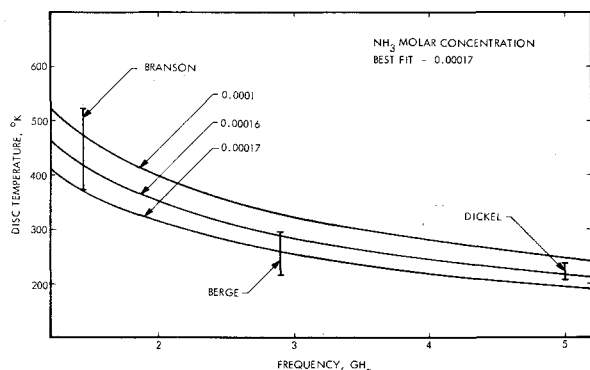


Fig. 18 Disk temperatures for the "Max Cool" temperature-pressure profile using different NH_3 molar concentrations.

1) Since there is much experimental evidence that the lapse rate of Jupiter's troposphere is adiabatic, only adiabatic lapse rates should be used in modeling Jupiter's troposphere.

2) The ammonia concentration for Jupiter's troposphere should be estimated from the brightness temperature data.

3) Since Eqs. (2, 3, and 6) are nonlinear, the easiest and most accurate method for determining the temperature-pressure-density probability distributions is the Monte Carlo method.

A detailed discussion of the implications and importance of the atmospheric models on entry probe design is beyond the scope of this paper. However, this Monte Carlo analysis suggests that a probe designed to withstand the large density and pressure uncertainties predicted in the NASA SP-8069 monograph illustrated in Figs. 13-16 would be overdesigned. Also, this analysis suggests that the transmitter and power associated with the probe communication system would be overdesigned if it was designed to withstand the NASA SP-8069 monograph.

Some possible mission planning implications for a Jupiter entry probe mission are represented in Fig. 17. If the mission objective was to design a probe to make *in situ* science measurements below the clouds then according to the NASA SP-8069 monograph the probe should be designed to survive to an atmospheric depth corresponding to a pressure of 40 atm;

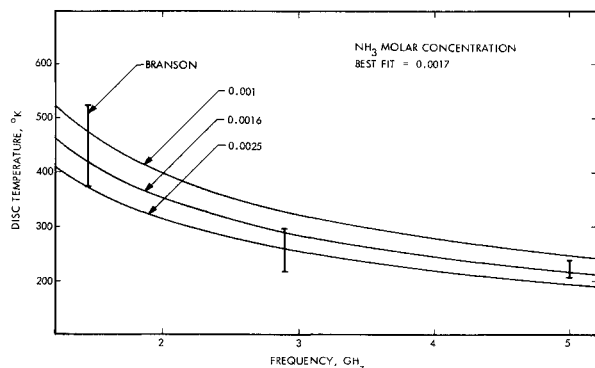
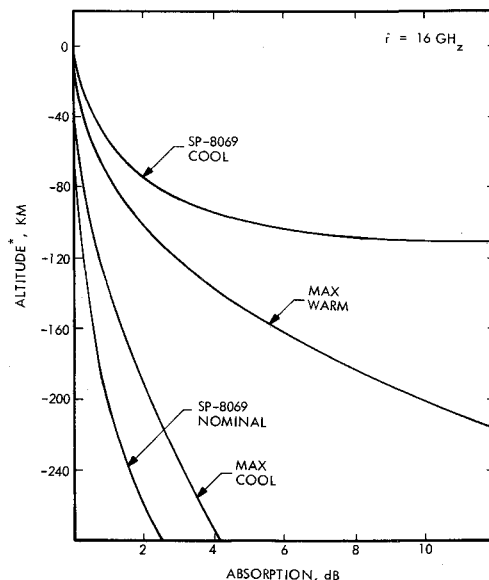


Fig. 19 Disk temperatures for the "Max Warm" temperature-pressure profile using different NH_3 molar concentrations.



*ZERO ALTITUDE AT ONE ATMOSPHERE

Fig. 20 Radio absorption in Jupiter's troposphere.

whereas, this analysis indicates that the probe would only need to be designed to survive to a depth corresponding to a pressure of 7 atm. Also, if a mission objective is to make science measurements within the ammonia clouds, then the science instruments need to be turned on before an atmospheric pressure of 0.5 atm is reached. If the NASA SP-8069 monograph models are used there is a possibility that the science instruments would not be turned on until the probe has passed through the ammonia clouds; thus, not accomplishing the hypothesized mission objective.

References

- Michaux, C. M., "Handbook of the Physical Properties of the Planet Jupiter," NASA SP-3031, 1967, pp. 87-112.
- Divine, T. N., "The Planet Jupiter (1970)," NASA SP-8069, Aug. 1971.
- Stone, P. H., "A Simplified Radiative-Dynamical Model for the Static Stability of Rotating Atmospheres," *Journal of the Atmospheric Sciences*, Vol. 29, No. 3, April 1972, pp. 405-418.
- Richardson, R. J., "Abundance of NH_3 on Jupiter Inferred from Microwave Radiometry Data," *Journal of Spacecraft and Rockets*, Vol. 10, No. 10, Oct. 1973, pp. 647-651.
- Lewis, J. S., "The Clouds of Jupiter and the NH_3 - H_2O and NH_3 - H_2S Systems," *Icarus*, Vol. 10, No. 3, May 1969, pp. 365-378.
- Trafton, L. M., "Model Atmospheres of the Major Planets," *Astrophysical Journal*, Vol. 147, No. 2, Feb. 1967, pp. 765-781.
- Westphal, J. A., "Observations of Localized 5-Micron Radiation from Jupiter," *Astrophysical Journal*, Vol. 157, No. 1, July 1969, pp. L63-L68.
- Ingersoll, A. P. and Cuzzi, J. N., "Dynamics of Jupiter's Cloud Bands," *Journal of the Atmospheric Sciences*, Vol. 26, No. 5, Sept. 1969, pp. 981-985.
- Divine, T. N., Private communication.
- Haudenschild, C., "Multi-phase Ammonia Water System," *JPL Space Programs Summary* 37-64, Vol. III.